

2 on Last Quiz w/ Divergence

AM:

$$\nabla \cdot \vec{F} = \nabla \cdot \langle y, -x, z \rangle = 0 + 0 + z = z,$$

$$\text{so } \oiint_{S_{\text{closed hemisphere}}} \vec{F} \cdot d\vec{S} = \iiint_{V_{\text{hemisphere}}} z \, dV \quad \left(= z \cdot \frac{1}{2} \cdot \frac{4\pi R^3}{3} \right. \begin{array}{l} \swarrow R=2 \\ \text{would} \\ \text{finish} \end{array} \left. = 32\pi/3 \right)$$

$$= 2 \int_{\rho=0}^2 \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/2} \rho^2 \sin\phi \, d\phi \, d\theta \, d\rho$$

$$= 4\pi \cdot \left(\frac{\rho^3}{3} \Big|_0^2 \right) \cdot \left(-\cos\phi \Big|_0^{\pi/2} \right) = 4\pi \cdot \frac{8}{3} \cdot (-0+1) = \boxed{\frac{32\pi}{3}}$$

PM:

$$\nabla \cdot \vec{F} = \nabla \cdot \langle 0, y, -z \rangle = 0 + 1 - 1 = 0,$$

$$\text{so } \oiint_S \vec{F} \cdot d\vec{S} = \iiint_{V_{\text{closed}}} 0 \, dV = \boxed{0}$$

[Since it's so easy, I graded extremely harshly, especially for the afternoon.]